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Estimation of Dynamic Rupture Parameters from the Radiated Seismic Energy and Apparent Stress

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Abstract. We propose a new procedure for estimating the critical slip weakening distance, D_c, by evaluating the spatio-temporal distribution of the apparent stress calculated from the spatio-temporal distribution of the slip velocity function on the fault plane obtained from a kinematic inversion of the earthquake source. The idea is based on the fact that the apparent stress can be related to the difference between the earthquake average stress and the frictional dynamical stress on the fault plane during the rupture process. From the cumulative slip - apparent stress relationship we estimate the critical slip, D_c, of the 1992 Landers earthquake. We find that the critical slip weakening plays an important role in controlling the rupture velocity. In the case of the Landers earthquake the very low rupture velocity in the central segment could be explained by the large D_c-value obtained for the main asperity on that segment.

Introduction

The stress conditions on faults before and during an earthquake and the inference of a constitutive friction law (characterizing the fault surface response to an applied stress) are the clue points in understanding the physics of the dynamic rupture phenomena of an earthquake from the beginning to the end. Simple models of earthquake stress release have been proposed in which the fault plane is initially at some shear stress value, σ_0, that drops to its final value, σ_1, at the end of the earthquake [*Orowan*, 1960; *Savage and Wood*, 1971]. During the slip event the fault motion is opposed by a stress associated with the dynamic friction, σ_f. The radiated energy, E_r, associated with this model can be expressed as follows:

$$E_r = \left[\frac{(\sigma_0 + \sigma_1)}{2} - \sigma_f\right] M_0/\mu \tag{1}$$

where M_0 is the seismic moment, σ_0, σ_1 and σ_f are initial, final and dynamic frictional stresses respectively, and μ is the rigidity. Equation (1) states that the radiated seismic energy is equal to the difference between the total energy released and the frictional energy.

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Calculation of the temporal evolution of the apparent stress

An expression for the total energy radiated by seismic waves, E_r, from a point source, taking into account the contribution of the S waves only (the P-waves contribution is negligible) can be obtained [*Rudnicki and Freund*, 1981] as follows:

$$E_R = \frac{1}{10\pi\rho\beta^3}\int_0^{T_{rup}} \ddot{M}(t)^2 dt \tag{2}$$

where T_{rup} is the source rupture duration, ρ is the density, β is the S-wave velocity, and $\ddot{M}(t)$ is the moment rate derivative. Using the definition of the apparent stress σ_{ap} (earthquake radiated energy normalized by the seismic moment) and equation (2), we obtain an expression for the temporal evolution of the apparent stress as the fault rupture proceeds:

$$\sigma_{ap}(t) = \frac{1}{10\pi\rho\beta^3}\frac{\int_0^t \ddot{M}(t)^2 dt}{M_0(t)} = \frac{1}{10\pi\rho\beta^3}\frac{\int_0^t \ddot{M}(t)^2 dt}{\int_0^t \dot{M}(t)} \tag{3}$$

where $M_o(t)$ is the cumulative moment up to some time t. A multiple-window kinematic inversion of the source gives information about the slip velocity at a large number of points across the fault surface. That information will allow us to compute the spatio-temporal evolution of the apparent stress in the fault plane.

Relationship between apparent stress and the fault friction law

From the energy balance of an earthquake *Madariaga* [1977] found an expression for the apparent stress in terms of the difference between the mean stress, $\bar{\sigma}_{ij}$ (average between initial and final stress) and the fault friction law at each point on the fault plane, $\sigma_{ij}(D)$:

$$\sigma_{ap}(D) = \frac{1}{DS_0}\int_{S_0} dS \int_0^D dD[\bar{\sigma}_{ij} - \sigma_{ij}(D)]n_j - \frac{\gamma}{D} \tag{4}$$

where D is the cumulative slip for the area S_0, γ is the fracture energy per unit advance of the rupture front and n_j is a unit normal vector. Let's assume that the stress release process is characterized by the slip weakening friction law [*Ida*, 1972] in which the shear stress drops from the yielding stress value, σ_y, to the final stress, σ_1, and the amount of slip required to get to σ_1 is the critical slip weakening distance (D_c). Under the above assumption the apparent stress obtained from the equation (4) will increases with increasing slip (shown schematically in figure 1) and its maximum

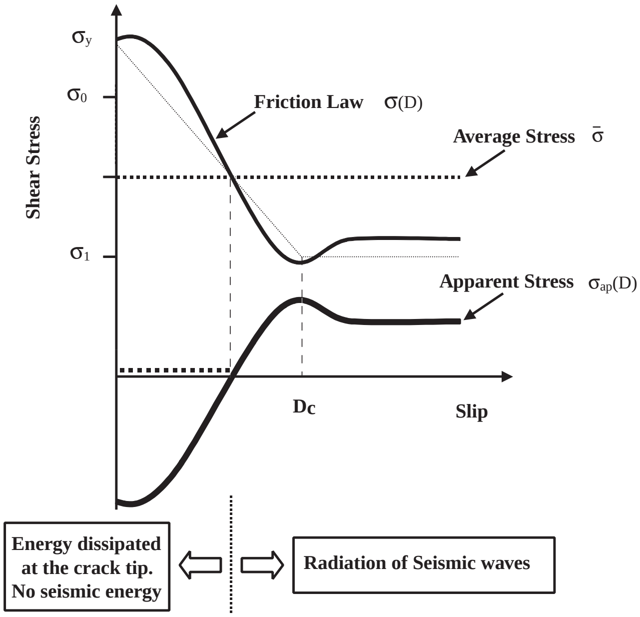

Figure 1. The apparent stress is equal to the difference between the average stress and the frictional dynamic stress. In this case a slip weakening friction Law is assumed.

value corresponds to the D_c value. Equation (4) therefore provides the physical basis to estimate D_c. As we can see from equation 4, the contribution to the seismic energy is negative for $\sigma(D) > \bar{\sigma}$, and positive for $\sigma(D) < \bar{\sigma}$. The negative term does not contribute to the radiation of seismic waves and should, perhaps, be incorporated into the fracture energy.

Estimation of the Critical Slip Weakening Distance (D_c)

Let's summarize the procedure for the estimation of the D_c-value in the following steps: (1) Perform a kinematic inversion of the source to obtain the slip velocity function at a large number of subfaults covering the fault plane surface; (2) Calculate the spatio-temporal distribution of the energy by applying equation (3) to the slip velocity function at each subfault across the fault plane; (3) Obtain the cumulative slip at each subfault from the slip velocity function; (4) Calculate the temporal evolution of the apparent stress by applying equation (2) to each subfault; (5) Plot the cumulative slip values vs the corresponding values of apparent stress obtained in steps 3 and 4, and read the value of slip corresponding to the maximum value of apparent stress. This value of slip will correspond to the D_c value as shown before.

Resolution of the estimation of the D_c values

The resolution criteria are related to the minimum D_c-value that we can resolve from a particular kinematic inversion. The minimum critical slip weakening resolvable from data at some particular point on the fault plane D_{cmin}, can be defined as the product of the average slip velocity ($\bar{\dot{s}}$) in the interval between the initiation of the rupture and the breakdown time T_c (time at which D_c is reached), and the minimum period resolvable from the data T_{cmin} [*Guatteri and Spudich*, 2000].

$$D_{cmin} = \bar{\dot{s}} T_{cmin} \tag{5}$$

The critical slip weakening (D_c) calculated from the temporal evolution of the apparent stress for each subfault should therefore be larger than the one in equation (5).

Calculation of the D_c value distribution for the 1992 Landers earthquake

We calculate the D_c value distribution for the Landers earthquake using the results of the kinematic source model by *Wald and Heaton* [1994]. The model consists of three fault segments aligned from south to north with lateral offsets. We calculate the temporal evolution of the apparent stress by applying equation (3) to all subfaults conforming the three segments of the kinematic model. In figure 2 we show the results of that calculation for selected subfaults on the central fault segment (Homestead Valley Fault, HV). From that figure we can see that the apparent stress increases with the rupture time for each subfault, particularly for the asperity region. We can also appreciate that the apparent stress increases with the increasing cumulative slip for each subfault (figure 3). These results are in agreement with the theoretical shape of the apparent stress deduced from equation (4) and shown in figure 1. The large initial peaks in figures 2 and 3 are explained by the fact that at the very beginning of the rupture the seismic moment is very small and the apparent stress will therefore be very large. However this problem can be avoided by an appropriate selection of the parametric source time function used in the kinematic inversion in such a way that the ratio $\frac{E_R(t)}{M_0(t)}$ tends to zero when the time approaches the rupture initiation.

We proceed to estimate D_c as shown by the arrows in figure 3, and obtain values between 2 m and 6 m for the HV fault segment. It is worth noting that most of the reliable estimates of D_c after applying the resolution criteria described before, corresponded to the HV fault segment. We find that the D_c-values are a large percentage of the total slip, by comparing the total slip and the D_c distribution for

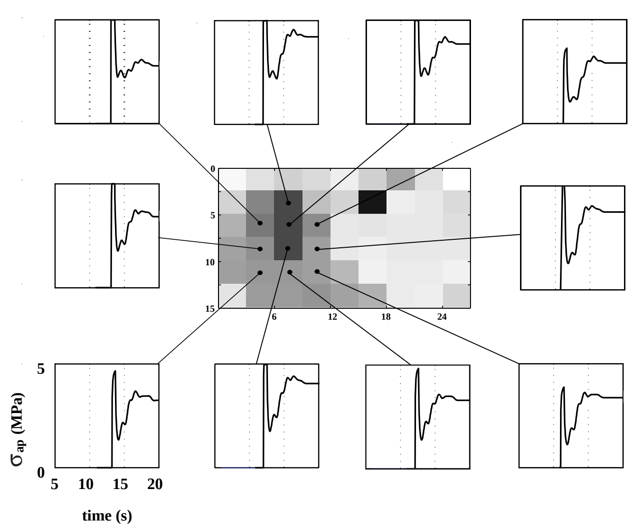

Figure 2. Temporal evolution of apparent stress for the Homestead Valley fault segment at selected subfaults.

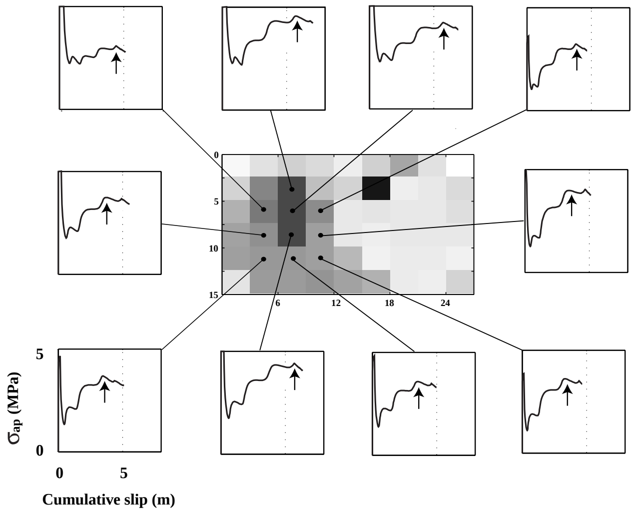

Figure 3. Cumulative slip - Apparent Stress relationship at selected subfaults for the Homestead Valley fault segment. The estimates of the D_c value are shown by an arrow.

the HV segment (figure 4a,b). The average D_c-value found for the HV segment was a 89% of the total slip. The average D_c-value found for all the segments together was 69% of the corresponding total slip. These large D_c-values could indicate that the fault requires a substantial amount of slip to break the barriers encountered during the propagation of the rupture. A similar result was obtained for a simple in-plane rupture model using a fracture mechanics approach [*Aki*, 1979].

Parametrization of the slip velocity function

The parametrization of the slip velocity function used in the source inversion procedure introduces a spurious frequency in the source time function that is related with the lack of overlapping time between two adjacent time windows, as in the case of *Wald and Heaton* [1994]. This is reflected on the shape of the apparent stress which can make difficult the estimation of D_c for some cases (figure 3). To make the estimation of D_c easier, the apparent stress was slightly smoothed by applying an averaging running window.

Calculation of the average D_c value

In the same way as we measured D_c for each subfault, we estimate the critical slip for the whole fault segment or for any particular region on the fault plane. We performed the calculation of the average D_c-value for each asperity on the fault plane. For that purpose we first calculate the moment rate function for the asperity region as follows:

$$\dot{M}(t) = \mu \sum_i \sum_j A_{ij} \dot{D}_{ij}(t) \tag{6}$$

where A_{ij} is the subfault area, and $\dot{D}_{ij}(t)$ is the slip velocity at some particular point (i, j) on the fault plane. By applying equation (6) we obtain the moment rate function of the asperity region by summing up the moment rate functions of all subfaults within that region (accounting for the rupture propagation). Following the above procedure we calculate the moment rate function for the three main asperities of the Landers earthquake. The duration of the rupture for each asperity is in average 7 sec (figure 5), and the main contribution to the total seismic energy is radiated from the asperity located in the Homestead Valley fault segment. We obtain values of D_c of 1m, 3.5 m and 1 m for the asperities located at the Johnson Valley (southern segment), Homestead Valley (central segment) and CampRock/Emerson (northern segment) fault segments respectively (figure 5).

Influence of the D_c value on the rupture velocity

The critical slip weakening distance is a very important factor in controlling the rupture velocity. The rupture velocity is controlled by a balance between the fracture energy and strain energy and by the ratio of the strength excess and the stress drop [*Fukuyama and Madariaga*, 1999; *Andrews*, 1976]. According to Wald and Heaton (1994) the Landers earthquake was characterized by an average rupture velocity of 3.6 km/sec at the beginning of the rupture, and remained approximately constant across the Johnson Valley Fault segment. For that region the average D_c-value found for the main asperity is 1 m. When the rupture entered to the Homestead Valley fault however, the rupture slowed down to a rupture velocity of about 1.0 km/sec to 1.4 km/sec. Interestingly, the value of D_c for the main asperity of that segment increased to an average value of 3.5m. Finally, the rupture speed up again to a value of 4 km/sec when it propagated to the CampRock/Emerson Fault segment, and the corresponding D_c-value for the main asperity of that segment reduced to a value of 1m. This observation is in agreement with the dynamic models of a spontaneous shear crack in which an increase in the fracture energy produced by an

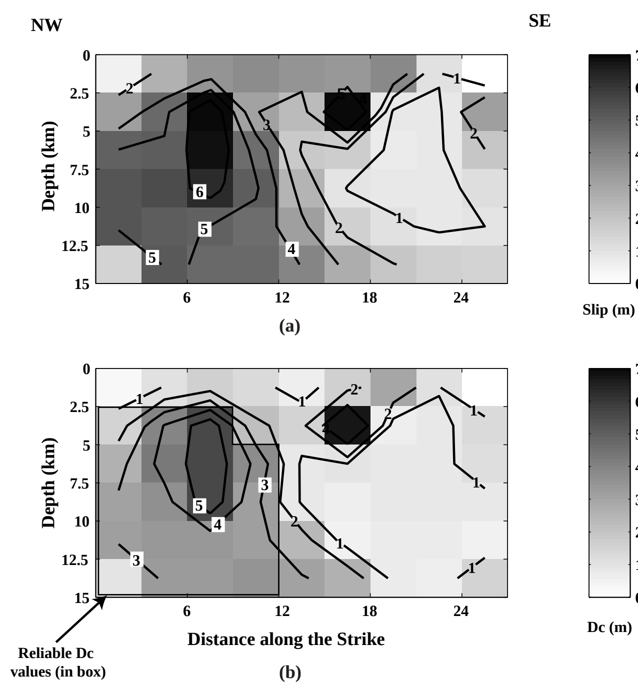

Figure 4. (a) Final Slip distribution of the Homestead Valley fault segment. (b) Distribution of the critical slip weakening for the Homestead Valley fault segment. The region corresponding to the values of D_c that satisfied the resolution criteria is indicated.

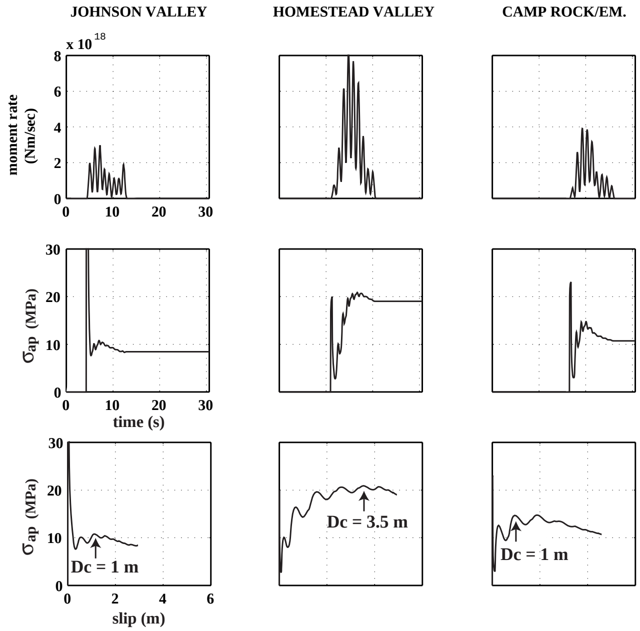

Figure 5. Calculation of the critical slip weakening distance for the main three asperities of the Landers earthquake. There is one assumed large asperity per fault segment.

increase in the D_c-value, implies a reduction in the rupture velocity and eventually rupture termination [*Fukuyama and Madariaga*, 1999]. The results presented show that the variation in rupture velocity during the Landers earthquake could be explained by a variation of the D_c value across the fault plane. In particular, our interpretation of the low value of rupture velocity at the Homestead Valley Fault segment comes from the high value of critical slip weakening found. On the other hand the rupture velocity of the Landers earthquake was not significantly impeded by presence of the lateral offsets, from the observation that those jumps were not characterized by a high value of strength excess compared with the rest of the fault which would slow down the rupture [*Bouchon et al.*, 1998].

Conclusions

We propose a new procedure to directly estimate the critical slip weakening distance, D_c, from results of a kinematic source inversion. Our values of D_c are, however, an upper bound estimate since we are using kinematic models that were determined in a low frequency band. Broadband frequency kinematic models of the source would allow more accurate estimates of D_c. The critical slip plays an important role in controlling the rupture velocity. In the case of Landers earthquake the very low rupture velocity in the central segment could be explained by the large D_c value obtained. On the other hand, small values of D_c corresponded to high values of rupture velocity. The results obtained suggest that the D_c value could be a substantial amount of the total slip in the fault.

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References

Aki, K., Characterization of Barriers on a Earthquake Fault, *J. Geophys. Res., 84,* 6140-6148, 1979.

Andrews, D. J., Rupture propagation with finite stress in antiplane strain, *J. Geophys. Res., 81,* 3575-3582, 1976.

Bouchon, M., M. Campillo, and F. Cotton, Stress field associated with the rupture of the 1992 Landers, California, earthquake and its implications concerning the fault strength at the onset of the earthquake, *J. Geophys. Res., 103,* 21091-21097, 1998.

Fukuyama, E. and R. Madariaga, Rupture Velocity on a Rectangular Fault (abstract), *Eos Trans. AGU, 80*(46), Fall Meet. Suppl., F754-F755, 1999.

Guatteri M. & P. Spudich, What Can Strong Motion Data Tell Us About Slip-Weakening Fault Friction Laws?, *Bull. Seismol. Soc. Am., 90,* 98-116, 2000.

Ida, Y., Cohesive Force across the Tip of a Longitudinal-Shear Crack and Griffith's Specific Surface Energy, *J. Geophys. Res., 77,* 3796-3805, 1972.

Madariaga, R., Implications of Stress-Drop models of Earthquakes for the Inversion of Stress Drop from Seismic Observations, Pageoph *115,* 301-316, 1977.

Orowan, E., Mechanism of seismic faulting in rock deformation, Geol. Soc. Am. Mem., *79,* 323-345, 1960.

Savage J., M. Wood, The relation between apparent stress and stress drop, *Bull. Seismol. Soc. Am., 61,* 1381-1388, 1971.

Rudnicki, J., and L. Freund, On energy radiation from seismic sources, *Bull. Seismol. Soc. Am., 71,* 583-595, 1981.

Wald, D. and T. Heaton, Spatial and Temporal Distribution of Slip for the 1992 Landers, California, Earthquake, *Bull. Seismol. Soc. Am., 84,* 668-691, 1994.

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